**Myths in tube circuit designs**

This online article complements “Myths in Tube Circuit Designs” published in audioXpress May 2023.

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**Contents**

[1 Introduction 1](#_Toc132020608)

[2 Determining distortion 2](#_Toc132020609)

[2.1 Power series 2](#_Toc132020610)

[2.2 The actual derivation 3](#_Toc132020611)

[3 Cancelation of even harmonics 6](#_Toc132020612)

[4 The triode model 8](#_Toc132020613)

[4.1 Why is the model of less value 10](#_Toc132020614)

[5 The influence of the ratio Ra/ri on the distortion 10](#_Toc132020615)

[6 References 12](#_Toc132020616)

# Introduction

In the May 2023 article, I made a statement about cancellation of even harmonics.

Also, some equations were given but not derived. I promised that I would provide that as supplemental material.

# Determining distortion

## Power series

Some readers may not be familiar with power series. Here is a brief introduction.

In Figure A2.1 you see a red line. It is the red graph of y = 5x . It is a straight line representing a perfect gain of 5. The blue line belongs to y = 5x – 0.01 x2, representing a perfect gain of 5 minus a small quadratic term. For small x-values the red line *approximates* the blue one.

The *quadratic* term is a *distortion*, in audio tube parlance. Terms with higher x-powers occur in practice, not only with tube circuitry, but in many technical situations.

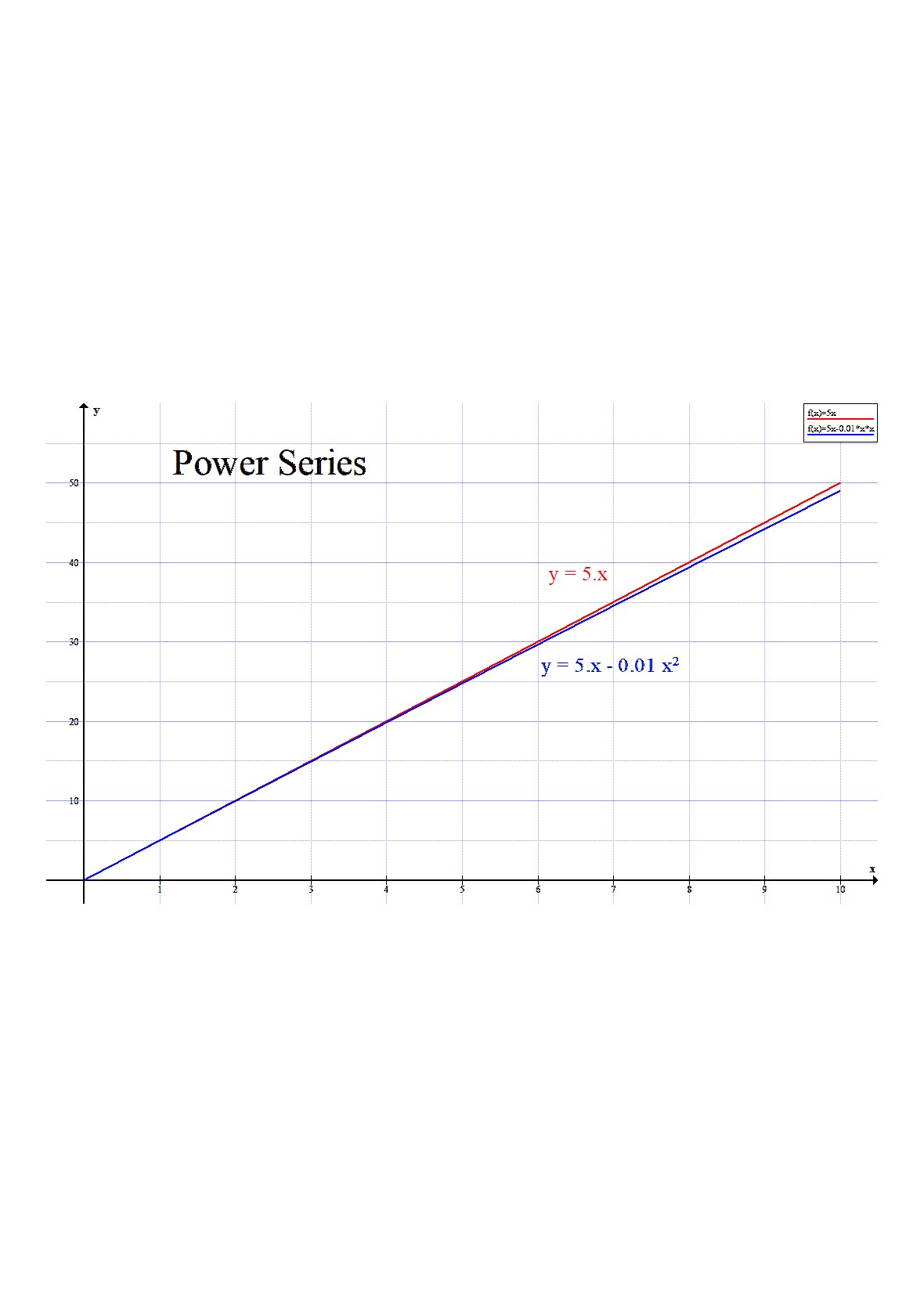


Figure A2.1 A linear function and a quadratic one

Please realize that for large values of x (say 1000 or more) the quadratic component dominates and the linear part can be neglected.

## The actual derivation

Most curves can be approximated by a polynomial with at most 5 terms.

The triode anode current can be expressed as a *power series* of the signal *v*:

(1)

We will offer a signal v = Vm cos ωt to the triode grid. Substituting this in the power series gives:

(2)

From goniometry we know:

(3)

and

(4)

Again, substituting these in the extended power series (2) gives:

(5)

After applying this cosine signal, the DC current I0 has increased with .

This is important, but not in the context of these derivations.

By definition the second harmonic (cos 2 ωt) is the ratio of its amplitude to the amplitude of the fundamental wave, which from (5) evaluates to:

(6)

For small signals we neglect the term with because in a triode this is almost zero. The distortion equation reduces to (7)

The distortion is linear proportional to the signal amplitude. Often, we see amplifier specs with extreme low distortion numbers. It means nothing when the corresponding voltages or powers are not mentioned.

Wanting to compute the distortion d2 we have two unknowns, α and β. We therefore need two equations. Applying two different signals will help us, when we look at the anode currents:

 (8)

 (9)

Adding these equations gives:

 (10)

Subtracting them we get:

 (11)

 (12)

We can now substitute (10) and (11) in (12). We then get:

(13)

In the nominator we find the difference of the anode signal positive and negative peak values. In the denominator we need the full peak to peak value.

Afbeelding met grafiek

Automatisch gegenereerde beschrijving

From the article: Figure 5 A slightly different design

In the above equation (13), the currents are used. We could just as well use the corresponding voltages, since . In tube’s characteristic graphs, the horizontal axis is usually more inches wide than the vertical one is high. Reading the voltages instead of the current gives greater accuracy.

In the original article Figure 5 the voltages belonging to i1 and i2 can be seen as:

v1 = 344-136.5 = 207.5 V and v2 = 487-344 = 143 V

The distortion by the second harmonic distortion is therefor:

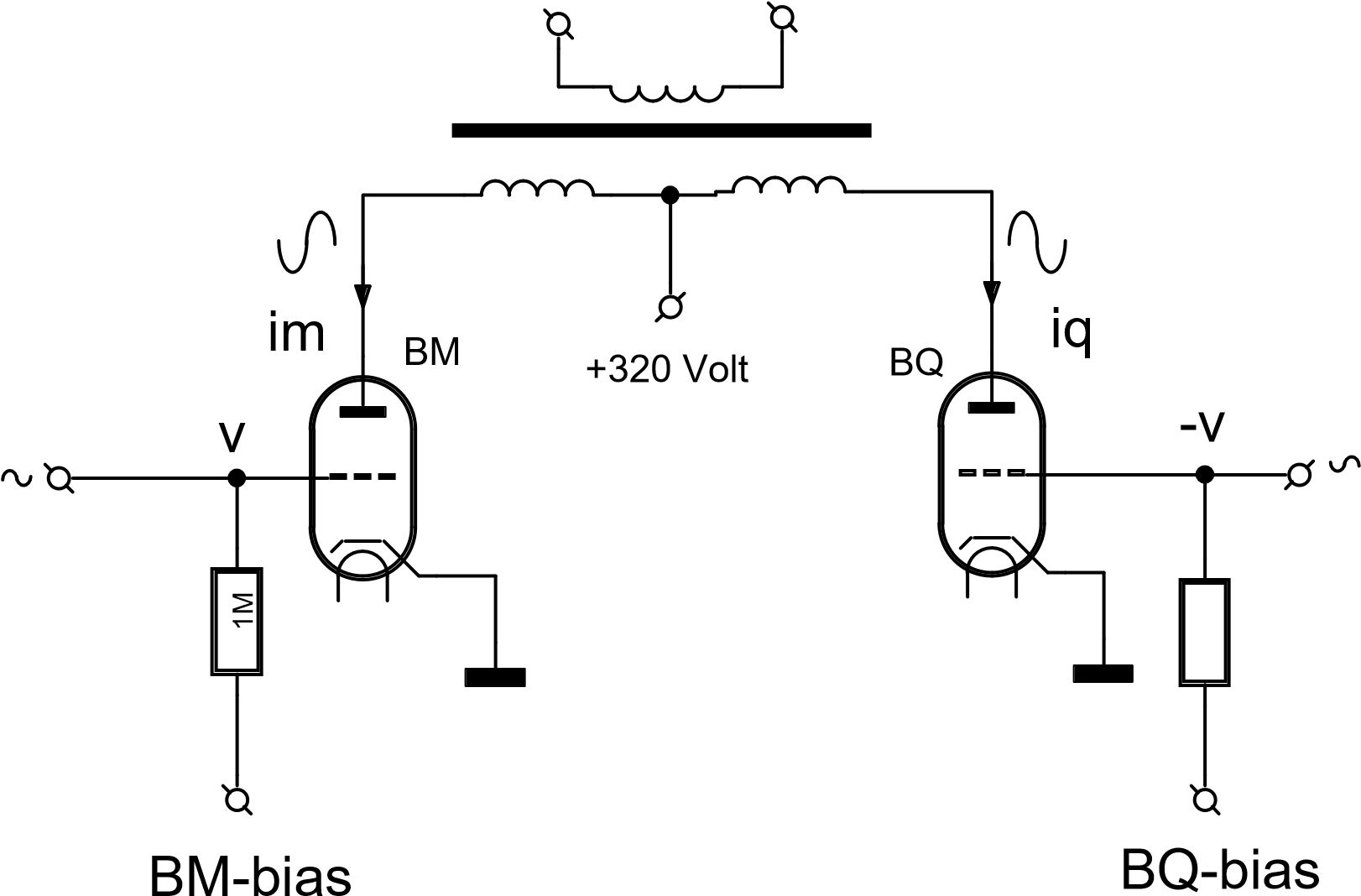
(14)

Note that this 9.2 % occurs when vaa = 350 Vpp. The distortion by the second harmonic is linearly proportional to the signal voltage as mentioned above.

# Cancelation of even harmonics

As was stated in the article, a push-pull transformer cancels out the even harmonics.

Here is the proof.



We use two triodes, M and Q. Tube M gets a signal v = cos ωt.

Tube Q gets a signal -cos ωt = cos (ωt + π)

This can be written as:

This is because cos (ωt + nπ) = -cos ωt , when n is odd; it is +cos ωt when n is even.

The magnetic field in the transformer’s core is the result of the difference of these two currents:

(15)

As you can see, for the distortion components. the *odd* harmonics *add up* and the *even* ones *cancel* each other out. You can consider the fundamental wave as the first odd harmonic and consider the DC current as the zeroth even harmonic. Because you already know that the DC currents cancel each other out, this will help you to remember which is which.

The moral of the story:

* triodes produce mainly even harmonics as distortions, and these are cancelled out.
* use a class A push-pull with triodes in your living room equipment.
* use pentodes in class B (producing third harmonics distortion) in guitar amps.

# The triode model

There is a mathematical model of a class A output triode. It is interesting, but I prefer working with the characteristics. These are much more accurate. The model was described in the world’s most famous Philips book on tubes [1] pages 90 .. 94. It was part of a series of 8 books. These books are ‘hors catégorie’ (French: Tour de France). It means ‘beyond categorization’. If only they were still available. At the end of my life, I will donate these books to a museum. Philips stated that there are German, French and English translations.

Here is the model. The derived results will be collected.

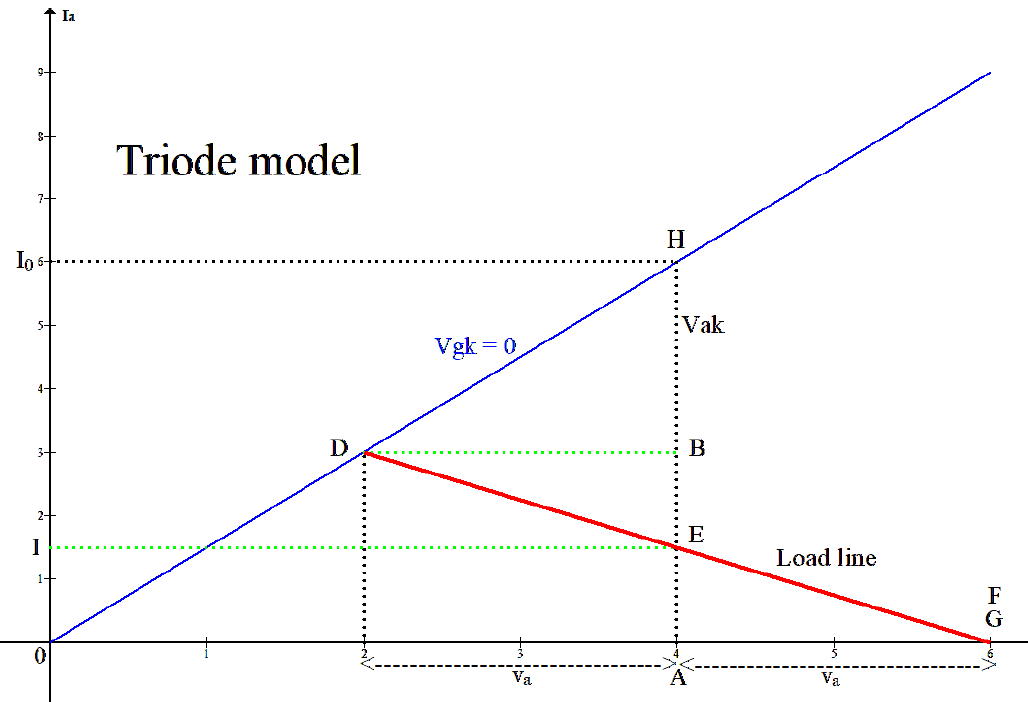


Figure A4.1 The idealized curves of a triode

The quiescent current is I. The supply voltage is Vak = Vb. These are the values at point E.

The maximum current is I0 (at point H,) which occurs when Vgk = 0.

In class A the current swings from I (point E) to 2 \* I (point B) and down to 0 (point A).

Of course we will want to have the maximum power, which will be reached when Vgk = 0. This is at point D. The power delivered to the loudspeakers then is : .

The factor ½ is because we are dealing with *peak values* and not with Vrms and Irms.

In figure A4.1 this va has the length BD. Triangles DBH and 0AH are congruent. Therefor

or and thus (result 1).

And thus (16)

This W reaches a maximum when or or

Substituting this gives the maximum power:

Also, and thus

The supplied power is . The efficiency is (result 2).

The transformers impedance is (result 3).

Up until now I did not pay attention to the tube maximum dissipation (the hyperbolic Wa = 29 W) in the original article figures 4 and 5).

When we want to allow this maximum dissipation, we take . Substituting this in (16) gives:

Substituting we get the final result (result 4).

So, the triode efficiency also has a maximum of 50 %, just like the a pentode. This contrasts with what is often published. We approach this maximum efficiency by taking a *high* value of Vak and a triode with a *low* Ri at the same time. Connection two triodes in parallel to get half the value of Ri does not solve a thing because the value of Pa between the two parentheses also doubles. The product of Pa and Ri remains the same.

We really need a triode with a low Ri by design.

The result as they are collected:

2. The efficiency is

## Why is the model of less value?

In the original article I concluded that the model was not that sound. Why is it not?

* With a triode we have real curves that are 3/2 powers. Approximating them with a straight line in A4.1 is too simple;
* The model neglects distortion. The swing at the right of point A is as long as the one to the left of point A. They both have the size va. Take a look at the characteristics as seen in the article Figures 4 and 5. You will see that the real half swings differ substantially. The right one is always shorter. This is the distortion.

# The influence of the ratio Ra/ri on the distortion

To understand this derivation, you must know about the rules of differentiating functions.

You may have learned this in an earlier education. If you did not, simply trust us 😊and happily use the results. Be alarmed, the derivation is complex.

Without an anode resistance, we can express the anode current in both a Taylor series and in a power series.

The Taylor series:

with (17)

The power series:

(18)

Combining them gives:

and and etcetera.

The derivatives here are a measure for the distortion components.

With an anode resistance Ra the function gets somewhat more complicated:

, where . We write this as .

Here also the derivatives G’(Vg), G’’(Vg), G’’’(Vg) etcetera are a measure for the distortion components, but now with the presence of an anode resistance.

These derivatives can be found by differentiating with respect to Vg. We must use the so called chain rule :

Regrouping we find (19)

You will get the second derivative by again differentiating with respect to Vg, or first to u and then multiplying with .

As is well known, the derivative of a fraction nominator / denominator is :

Elaborating this further and substituting (19) gives :

(20)

The first derivative is, according (19) divided by a factor

The second derivative is, according (20) divided by a factor

The distortion is thus divided by a factor

In this equation we have because , according to (18).

The conclusion is:

Further suffering gives

In these equations d2 and d3 are the original distortions by respectively the second and third harmonics. They are large because we deal with real triode curves. They are the results of the triode 3/2 power law. The left-hand numbers with an accent indicate the distortions resulting *after* applying an anode resistor Ra. These distortions can be significantly lower.

# References

[1] Toepassing van de electronenbuis. (applications of the electronic tube).

Philips, dr B.G. Dammers et al. 1951